ATME COLLEGE OF ENGINEERING

13th KM Stone, Bannur Road, Mysore - 570 028

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

NOTES

Course: Power System Analysis-2

Course Code: BEE703

SEMESTER: VII

INSTITUTIONAL VISION AND MISSION

VISION:

- Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

MISSION:

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torchbearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence.

Department Vision and Mission

Vision:

To create Electrical and Electronics Engineers who excel to be technically competent and fulfill the cultural and social aspirations of the society.

Mission:

- To provide knowledge to students that builds a strong foundation in the basic principles of electrical engineering, problem solving abilities, analytical skills, soft skills and communication skills for their overall development.
- To offer outcome based technical education.
- To encourage faculty in training & development and to offer consultancy through research & industry interaction.

Program Educational Objectives (PEOs)

PEO1:

To produce Electrical and Electronics Engineers who will exhibit the technical and managerial skills with professional ethics for the societal progress.

PEO2:

To make Graduates continuously acquire, enhance their technical and socio-economic skills and also to be globally competent.

PEO3:

To impart the experience of research and development to Graduates so that they develop abilities in offering solutions to relevant diverse career path.

PEO4:

To produce quality engineers with a team leading capabilities, also show good coordination to contribute towards real time application of projects

Program Outcomes (POs)

Engineering Graduates will be able to:

PO1: Engineering Knowledge: Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3: Design / Development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4: Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5: Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6: The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7: Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8: Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9: Individual and team work: Function effectively as an individual and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11: Project management and finance: Demonstrate knowledge and understanding of the engineering management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12: Life-long learning: Recognize the need for and have the preparation and ability to engage in independent and lifelong learning in the broadest context of technological change.

Program Specific Outcomes (PSOs)

The students will develop an ability to produce the following engineering traits:

PSO1: Apply the concepts of Electrical & Electronics Engineering to evaluate the performance of power systems and also to control industrial drives using power electronics.

PSO2: Demonstrate the concepts of process control for Industrial Automation, design models for environmental and social concerns and also exhibit continuous self- learning.

Module-1

Network Topology

Introduction

An important step in the procedure for solving any circuit problem consists first in selecting a number of independent branch currents as (known as loop currents or mesh currents) variables, and then to express all branch currents as functions of the chosen set of branch currents. Alternately a number of independent node pair voltages may be selected as variables and then express all existing node pair voltages in terms of these selected variables. For simple networks involving a few elements, there is no difficulty in selecting the independent branch currents or the independent node-pair voltages. The set of linearly independent equations can be written by inspection. However for large scale networks particularly modern electronic circuits such as integrated circuits and microcircuits with a larger number of inter-connected branches, it is almost impossible to write a set of linearly independent equations by inspection or by mere intuition. The problem becomes quite difficult and complex. A systematic and step by step method is therefore required to deal with such networks. Network topology(graph theory approach) is used for this purpose. By this method, a set of linearly independent loop or node equations can be written in a form that is suitable for a computer solution.

Terms and definitions

The description of networks in terms of their geometry is referred to as network topology. The adequacy of a set of equations for analysing a network is more easily determined topologically than algebraically.

Graph (or linear graph):

A network graph is a network in which all nodes and loops are retained but its branches are represented by lines. The voltage sources are replaced by short circuits and current sources are replaced by open circuits. (Sources without internal impedances or admittances can also be treated in the same way because they can be shifted to other branches by E-shift and/or I-shift operations.)

Branch: A line segment replacing one or more network elements that are connected in series or parallel.

Node: Interconnection of two or more branches. It is a terminal of a branch. Usually interconnections of three or more branches are nodes.

Path: A set of branches that may be traversed in an order without passing through the same node more than once.

Loop: Any closed contour selected in a graph.

Mesh: A loop which does not contain any other loop within it.

Planar graph: A graph which may be drawn on a plane surface in such a way that no branch passes over any other branch.

Non-planar graph: Any graph which is not planar.

Oriented graph: When a direction to each branch of a graph is assigned, the resulting graph is called an oriented graph or a directed graph.

Connected graph: A graph is connected if and only if there is a path between every pair of nodes.

Sub graph: Any subset of branches of the graph.

Tree: A connected sub-graph containing all nodes of a graph but no closed path. i.e. it is a set of branches of graph which contains no loop but connects every node to every other node not necessarily directly. A number of different trees can be drawn for a given graph.

Link: A branch of the graph which does not belong to the particular tree under consideration. The links form a sub-graph not necessarily connected and is called the co-tree.

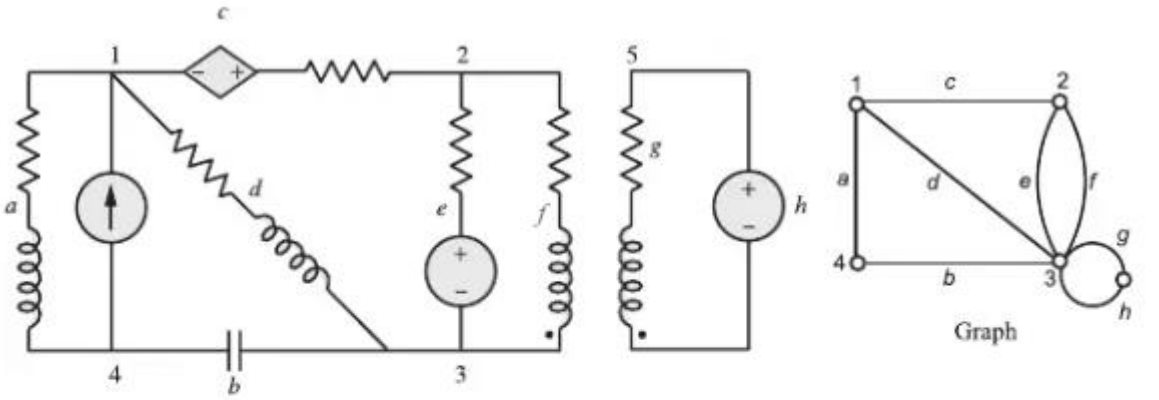
Tree compliment: Totality of links i.e. Co-tree.

Independent loop: The addition of each link to a tree, one at a time, results one closed path called an independent loop. Such a loop contains only one link and other tree branches. Obviously, the number of such independent loops equals the number of links.

Tie set: A set of branches contained in a loop such that each loop contains one link and the remainder are tree branches.

Tree branch voltages: The branch voltages may be separated in to tree branch voltages and link voltages. The tree branches connect all the nodes. Therefore if the tree branch voltages are forced to be zero, then all the node potentials become coincident and hence all branch voltages are forced to be zero. As the act of setting only the tree branch voltages to zero forces all voltages in the network to be zero, it must be possible to express all the link voltages uniquely in terms of tree branch voltages. Thus tree branch form an independent set of equations.

Cut set: A set of elements of the graph that dissociates it into two main portions of a network such that replacing any one element will destroy this property. It is a set of branches that if removed divides a connected graph in to two connected sub-graphs. Each cut set contains one tree branch and the remaining being links. Fig. shows a typical network with its graph, oriented graph, a tree, co-tree and a non-planar graph


c
1
2
a
d
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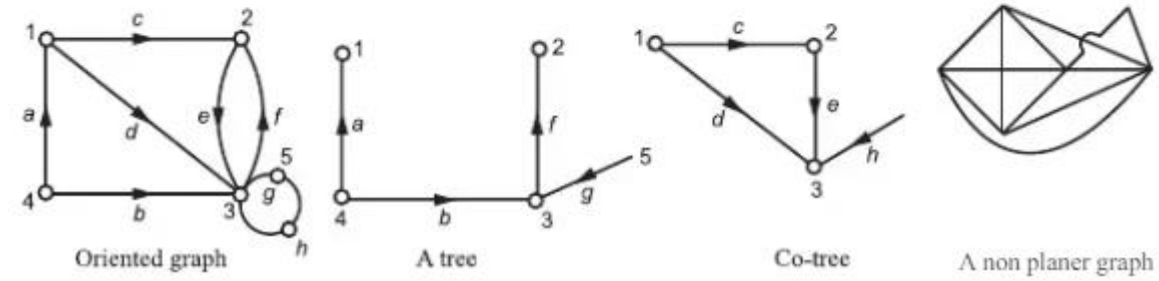
Network
Graph


Oriented graph
A tree
Co-tree
A non planer graph

LOAD FLOW STUDIES

REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. Solution Linear equations: *
Direct methods:
- Cramer‟ s (Determinant) Method,
- Gauss Elimination Method (only for smaller systems), - LU Factorization (more preferred method), etc.

* **Iterative methods**: -
Gauss Method
- Gauss-Siedel Method (for diagonally dominant systems)

3. Solution of Nonlinear equations:
Iterative methods only:
- Gauss-Siedel Method (for smaller systems)
- Newton-Raphson Method (if corrections for variables are small)

4. Solution of differential equations:
Iterative methods only:
- Euler and Modified Euler method,
- RK IV-order method,
- Milne‟ s predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:
_ Selection of initial solution/ estimates
_ Determination of fresh/ new estimates during each iteration _
Selection of number of iterations as per tolerance limit
_ Time per iteration and total time of solution as per the solution method selected _ Convergence and divergence criteria of the iterative solution _
Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:
In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate. The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.The Newton-Raphson method is very much useful for solution of non – linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:

_ The Kirchhoff‟ s relations holding good,
_ Capability limits of reactive power sources,
_ Tap-setting range of tap-changing transformers,
_ Specified power interchange between interconnected systems,
_ Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00 , as per flat-start procedure of iterative solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of YBUS based LFA:

The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and

hence only n(n+1)/2 elements of YBUS need to be stored for a n-bus system. Further, in the YBUS matrix, Yij = 0, if an incident element is not present in the system connecting the buses „i‟ and „j‟ . since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, YBUS of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{th} \text{ order:} = \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$\mathbf{S} = (\mathbf{Z} / \mathbf{n^2}) \times \mathbf{100}\ \% \qquad \mathbf{(1)}$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of YBUS is extensively used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, it‟s inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$S_i = S_{Gi} - S_{Di}$ **(2)**

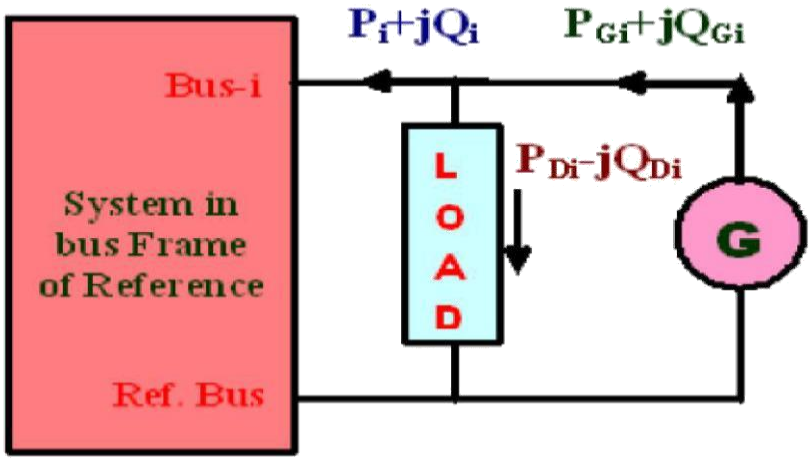


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

Si = _Sij " i = 1, 2, ……..n
(3)

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2, ……..n **(4)** where IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS
(5)

where

$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix}$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$S_i = V_i\ I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\ V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \qquad \textbf{(7)}$$

Let $V_i \triangleq |V_i| \angle \delta_i = |V_i| (\cos\ \delta_i + j \sin\ \delta_i)$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (\cos \delta_{ij} + j \sin \delta_{ij}) \, (G_{ij} - j B_{ij}) \qquad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \qquad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \qquad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form as

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} \qquad \textbf{(11)}$$

Again, we get from (7),

$$S_i = |V_i| \angle \delta_i \sum_{j=1}^{n} |Y_{ij}| \angle -\theta_{ij} \, |V_j| \angle -\delta_j \qquad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos(-\theta_{ij} + \delta_i - \delta_j)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall \; i = 1, 2, \ldots\ldots\ldots n, \qquad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \sin -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i| \, |V_j| \, |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall \; i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the „power flow equations" or the „load flow equations" in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-*n*, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0o), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus *i*, the data required includes the bus number, active power generation PGi, the specified voltage magnitude $i\ sp\ V$, , minimum reactive power limit Qi,min, and maximum reactive power limit Qi,max.

Load data: For all loads the data required includes the the bus number, active power demand PDi, and the reactive power demand QDi.

Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the starting bus number *i,* ending bus number *k,.r*esistance of the line, reactance of the line and the half line charging admittance.

Transformer data:
For every transformer connected between buses *i* and *k* the data to be given includes: the starting bus number *i*, ending bus number *k*, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio *a.*

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance (Gsh + j Bsh).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} \; V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} \, V_j \right) \qquad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} \, V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} \; V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} \; V_j \qquad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall \; i = 2, 3, \ldots\ldots\ldots n \qquad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.00\angle 0^{0}$. This is normally refered as the ***flat start*** solution. 4. Update the voltages. In any $(k+1)st$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \quad \forall\ i=2,3,\ldots n \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st* iteration. Hence these values are used. For buses *(i+1).....n,* values from previous, *kth* iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall\ i = 2,3,\ldots n \qquad \textbf{(19)}$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\,\mathrm{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij}\, V_j\right\}$$

Where *Im* stands for the imaginary part. At any $(k+1)^{st}$ iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\,\mathrm{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij}\, V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij}\, V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}|\,\angle\delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $(k+1)$ *i* Q computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows:

If $Q_i < Q_{i,min}$ Then $Q_i = Q_{i,min}$.

If $Q_i > Q_{i,max}$ Then $Q_i = Q_{i,max}$.

(23)

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad (24)$$

where is a real number. When =1, the value of $(k+1)$ is the computed value. If 1<<2 then α the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if* $V_1 = 1\angle 0^0$ *pu.*

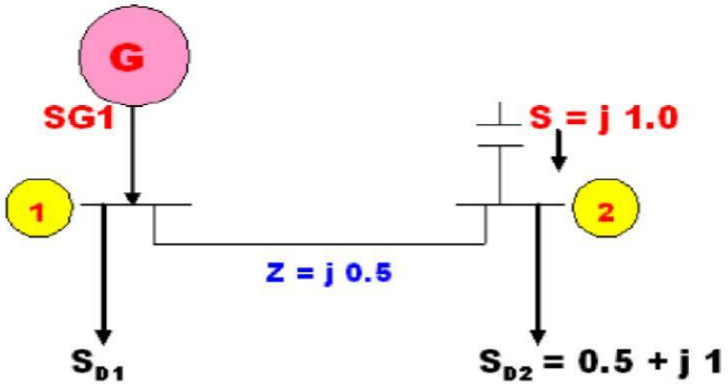


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\ 1.0) = -0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\ 0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 1.0 - \text{j}0.25 = 1.030776 \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.94118 - \text{j}\ 0.23529 = 0.970145 \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.9375 - \text{j}\ 0.249999 = 0.970261 \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933612 - \text{j}\ 0.248963 = 0.966237 \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933335 - \text{j}\ 0.25 = 0.966237 \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10-6 pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472\ \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472\ \angle\ 14.931^0$$

$$= 0.5 + \text{j}\ 0.133329\ \text{pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

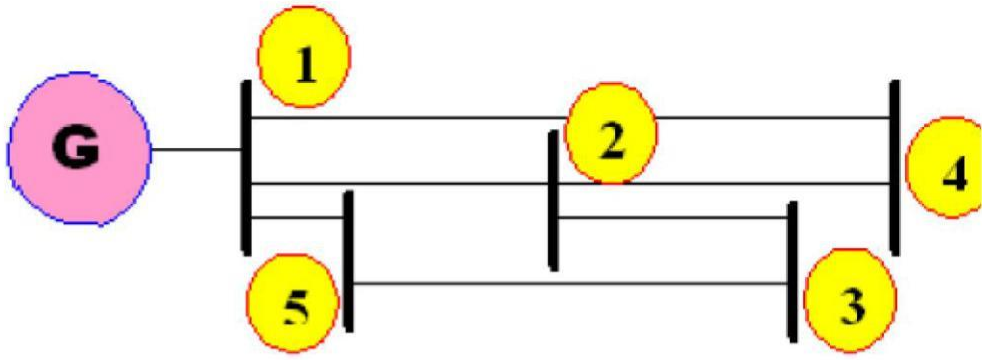
$$= 0.517472\ \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -0.5 + \text{j}\ 0.0\ \text{pu}$$

The total loss in the line is given by S12 + S21 = j 0.133329 pu Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP}\rvert$ (pu)	δ
1	-	-	-	-	1.02	0^o
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be $V_1^0 = 1.02+j0.0$ in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294)\times 1.0\angle 0^0\}\right]$$

$$= 0.98140\angle -3.0665^\circ = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^\circ$; $\delta_2 = -3.0665^\circ$; $\delta_3 = 0^\circ$; $\delta_4 = 0^\circ$ and $\delta_5 = 0^\circ$

$\therefore\ \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^\circ$ ($\delta_{ik} = \delta_i - \delta_k$); $\delta_{32} = 3.0665^\circ$

$Q_3 = 1.04\,[1.02\,(0.0+j0.0) + 0.9814\,\{-1.17647 \times \sin(3.0665^\circ) - 4.70588$

$\times\cos(3.0665^\circ)\} + 1.04\{-9.41176 \times\cos(0^\circ)\} + 1.0\,\{0.0 + j0.0\} + 1.0\{-4.70588\times\cos(0^0)\}]$

$= 1.04\,[-4.6735 + 9.78823 - 4.70588] = 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \left\{(-1.7647 + j4.70588)\times(0.98140\angle -3.0665^{o})\right\}\right.$$

$$\left. - \left\{(-1.17647 + j4.70588)\times(1\angle 0^{o})\right\}\right]$$

$$= 1.05569\angle 3.077^{o} = 1.0541 + \text{j}0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^{0}$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^{o} - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \left\{(-0.39215 + j1.56862)\times 1.02\angle 0^{o}\right\}\right.$$

$$\left. - \left\{(-0.58823 + j2.35294)\times(0.98140\angle -3.0665^{o})\right\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^{o} \text{ pu} = 0.94796 - \text{j}0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^{o} - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \left\{(-1.17647 + j4.70588)\times 1.02\angle 0^{o}\right\}\right.$$

$$\left. - \left\{(-1.17647 + j4.70588)\times 1.04\angle 3.077^{o}\right\}\right]$$

$$= 0.994618\angle -1.56^{o} = 0.994249 - \text{j}0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^{0}$ pu $\qquad V_2 = 0.98140\ \angle -3.066^{0}$ pu

$V_3 = 1.04\angle 3.077^{0}$ pu $\qquad V_4 = 0.955715\angle -7.303^{0}$ pu

and $\qquad V_5 = 0.994618\angle -1.56^{0}$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses

(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25_Q2_1.0 pu.

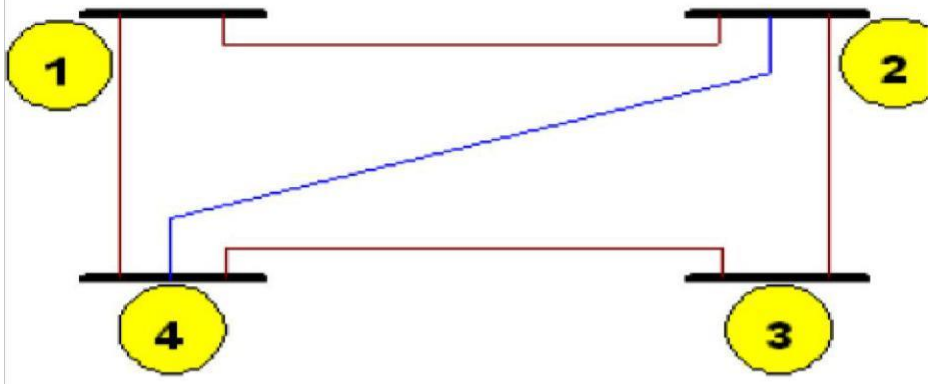


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04 \angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$Y_{BUS} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0 \\ -2.0 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2.0 + j6.0 \\ 0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0 \angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21} V_1^o - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.02014\angle 2.605^o$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}\,V_1^o - Y_{32}\,V_2^1 - Y_{34}\,V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.03108\angle -4.831^o$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}\,V_1^o - Y_{42}\,V_2^1 - Y_{43}\,V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^o)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^o)\}\right]$$

$$= 1.02467\angle -0.51^o$$

Hence

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.02014\ \angle 2.605^0$ pu

$V_3^1 = 1.03108\ \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467\angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\left[|V_1|\left(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}\right) + |V_2|\left(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22}\right)\right.$$
$$\left. + |V_3|\left(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}\right) + |V_4|\left(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24}\right)\right]$$

$$= 1.04\,[1.04\,\{-6.0\} + 1.04\,\{11.0\} + 1.0\{-2.0\} + 1.0\,\{-3.0\} = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \left\{(-2.0 + j6.0)\times\left(1.04\angle 0^o\right)\right\}\right.$$
$$\left. - \left\{(-0.666 + j2.0)\times\left(1.0\angle 0^o\right)\right\} - \left\{(-1.0 + j3.0)\times\left(1.0\angle 0^0\right)\right\}\right]$$

$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04\angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\angle 0^0} - \left\{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\right\}\right.$$
$$\left. - \left\{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\right\} - \left\{(-2.0 + j6.0)\times\left(1.0\angle 0^0\right)\right\}\right]$$

$$= 1.035587\angle -4.951^o \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \left\{(-1.0 + j3.0)\times\left(1.04\angle 1.846^o\right)\right\}\right.$$
$$\left. - \left\{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\right\}\right]$$

$$= 0.9985\angle -0.178^o$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $V_4^1 = 0.9985\angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \leq Q_2 \leq 1$ pu. If $0.25 \leq Q_2 \leq 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

Limitations of GS load flow analysis

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

3.7

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of *n* non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \tag{25}$$

Let $x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots\ldots x_n^{\,0}$, be the initial guess of unknown variables and $\Delta x_1^{\,0}, \Delta x_2^{\,0} \ldots\ldots \Delta x_n^{\,0}$ be the respective corrections. Therefore,

$$f_i(x_1^{\,0} + \Delta x_1^{\,0}, x_2^{\,0} + \Delta x_2^{\,0} \ldots\ldots\ldots x_n^{\,0} + \Delta x_n^{\,0}) = 0 \qquad i = 1,2\ldots\ldots n \tag{26}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^{\,0}, x_2^{\,0} \ldots\ldots x_n^{\,0}) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^{\,0} + \left(\frac{\partial f_i}{\partial x_2}\right) \Delta x_2^{\,0} + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right) \Delta x_n^{\,0} \right]$$

$$+ \text{Higher order terms} = 0 \qquad \forall\ i = 1,2\ldots\ldots n \tag{27}$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at $(x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots\ldots x_0^{\,n})$. If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^{\,0} \\ f_2^{\,0} \\ . \\ . \\ . \\ f_n^{\,0} \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & . \; . \; . & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & . \; . \; . & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ . & . & . \; . \; . & . \\ . & . & . \; . \; . & . \\ . & . & . \; . \; . & . \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & . \; . \; . & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^{\,0} \\ \Delta x_2^{\,0} \\ . \\ . \\ . \\ \Delta x_n^{\,0} \end{bmatrix} = 0 \tag{28}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $$F^0 = -J^0 \Delta X^0$$

Or $$\Delta X^0 = -[J^0]^{-1} F^0 \tag{29}$$

And $$X^1 = X^0 + \Delta X^0 \tag{30}$$

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ...x_n) = 0$, where $x_1, x_2, ...x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \qquad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \qquad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad \textbf{(33)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \quad \textbf{(34)}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \quad \textbf{(35)}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J_3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

3. 8

ALGORITHM FOR NR METHOD

IN POLAR COORDINATES

1. Formulate the Y_{BUS}

2. Assume initial voltages as follows:

$$V_i = |V_{i,sp}|\angle 0^0 \quad \text{(at all PV buses)}$$

$$V_i = 1\angle 0^0 \quad \text{(at all PQ buses)}$$

3. At $(r+1)^{st}$ iteration, calculate $P_i^{(r+1)}$ at all the PV and PQ buses and $Q_i^{(r+1)}$ at all the PQ buses, using voltages from previous iteration, $V_i^{(r)}$. The formulae to be used are

$$P_{i,Cal} = P_i = G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_{i,Cal} = Q_i = \quad -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

4. Calculate the power mismatches (power residues)

$$\Delta P_i^{(r)} = P_{i,sp} - P_{i,cal}^{(r+1)} \quad \text{(at PV and PQ buses)}$$

$$\Delta Q_i^{(r)} = Q_{i,sp} - Q_{i,cal}^{(r+1)} \quad \text{(at PQ buses)}$$

5. Calculate the Jacobian $[J^{(r)}]$ using $V_i^{(r)}$ and its elements spread over H, N, M, L sub- matrices using the relations derived as in (36).

6. Compute

$$\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V|} \end{bmatrix} = [J^{(r)}]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows:

$$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta\delta_i^{(r)} \quad \text{(at all buses)}$$

$$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta|V_i|^{(r)}$$

8. Go to step 3 and iterate till the power mismatches are within acceptable tolerance.

DECOUPLED LOAD FLOW

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \qquad \textbf{(37)}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $|V^{(r+1)}| = |V^{(r)}| + |\Delta V^{(r)}|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

The matrix (37) reduces to

$$[\Delta P] = [|V_i||V_j|B'_{ij}][\Delta\delta]$$

$$[\Delta Q] = [|V_i||V_j|B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(38)}$$

Where B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = [B'_{ij}][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(39)}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

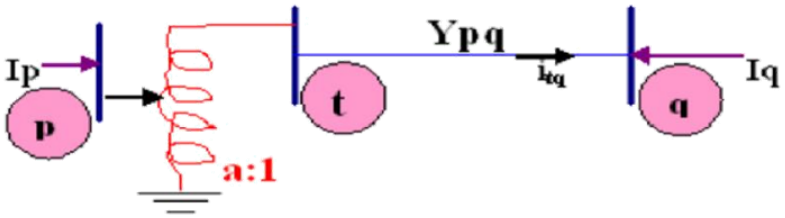


Fig. 2. Equivalent circuit of a tap setting transformer

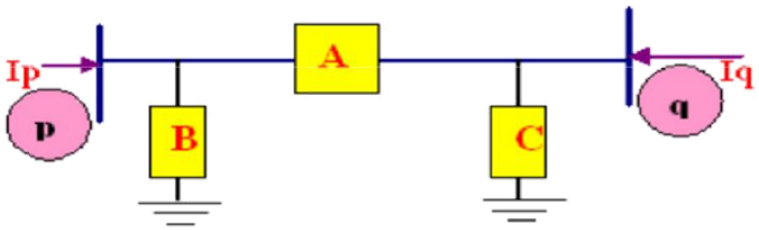


Fig. 3. π-Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the π-equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

$A = Ypq$

$B = (1/a - 1)(1/a + 1 - Eq/Ep)\, Ypq$

$C = (1 - 1/a)(Ep/Eq)\, Ypq$

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if

the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

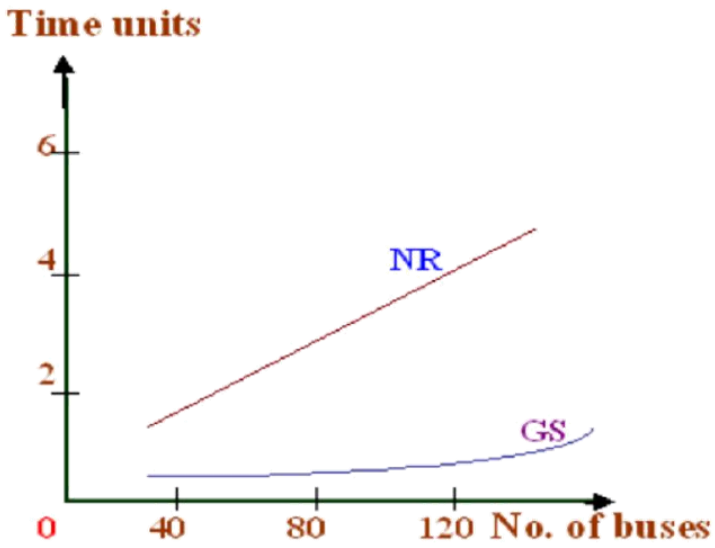


Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

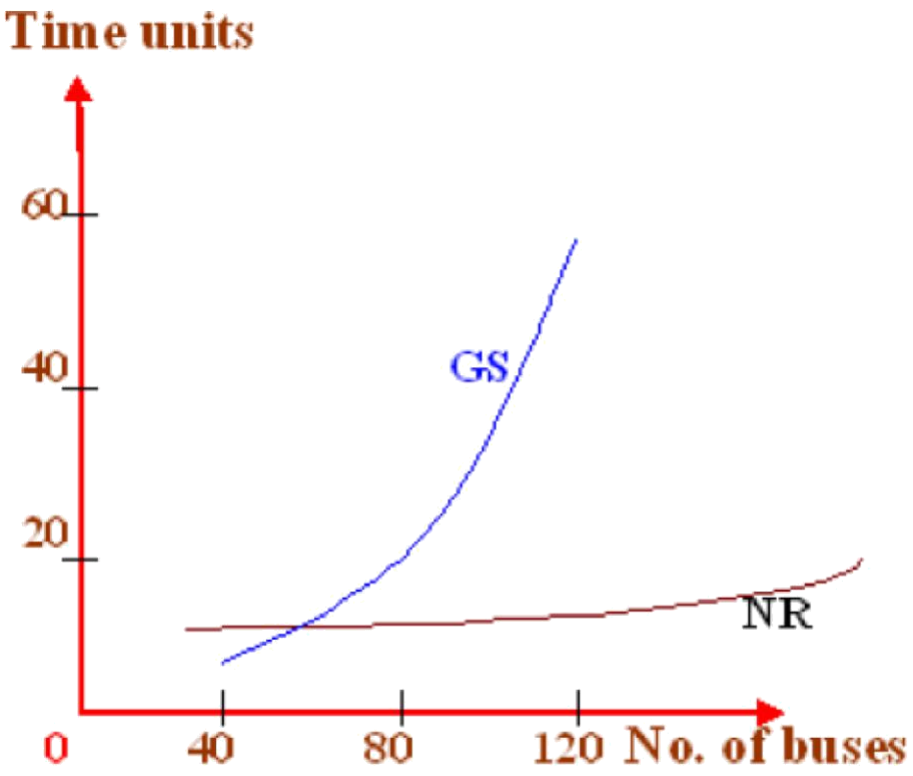


Figure 5. Total time of Iteration in GS and NR methods

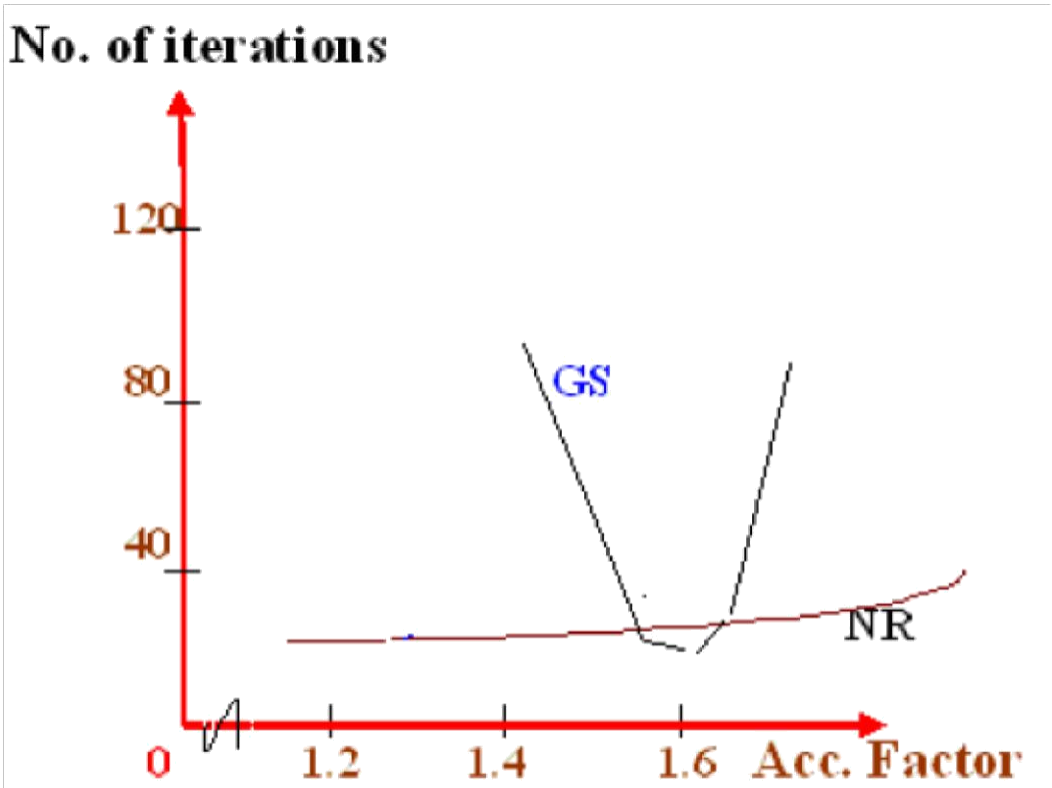


Figure 6. Influence of acceleration factor on load flow methods

FINAL WORD

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular. The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.

1. Using generalized algorithm expressions for each case of analysis, explain the load flow studies procedure, as per the G-S method for power system having PQ and PV buses, with reactive power generations constraints.
2. Derive the expression in polar form for the typical diagonal elements of the sub matrices of the Jacobian in NR method of load flow analysis.
3. Compare NR and GS method for load flow analysis procedure in respect of the following i) Time per iteration ii) total solution time iii) acceleration factor iv)number of iterations
4. Explain briefly fast decoupled load flow solution method for solving the non linear load flow equations.
5. Draw the flow chart of NR method for load flow analysis.
6. Explain the representation of transformer with fixed tap changing during the load flow studies
7. What are the assumptions made in fast decoupled load flow method? Explain the algorithm briefly, through a flow chart.
8. Explain the NR load flow method? Explain the algorithm briefly, through a flow chart.
9. What is load flow analysis? What is the data required to conduct load flow analysis? Explain how buses are classified to carry out load flow analysis in power system. What is the significance of slack bus.
10. The following is the system data for a load flow analysis. The line data are shown and also real and reactive powers data is given. Determine the voltage at the end of first iteration using GS method. Take $\alpha=1.6$

ECONOMIC OPEARATION OF POWER SYSTEM

INTRODUCTION

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants. The operation economics can again be subdivided into two parts.

i) Problem of *economic dispatch*, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.

ii) Problem of *optimal power flow*, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

During operation of the plant, a generator may be in one of the following states: i) Base supply without regulation: the output is a constant.

ii) Base supply with regulation: output power is regulated based on system load. iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes. iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

PERFORMANCE CURVES

INPUT-OUTPUT CURVE

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig1.

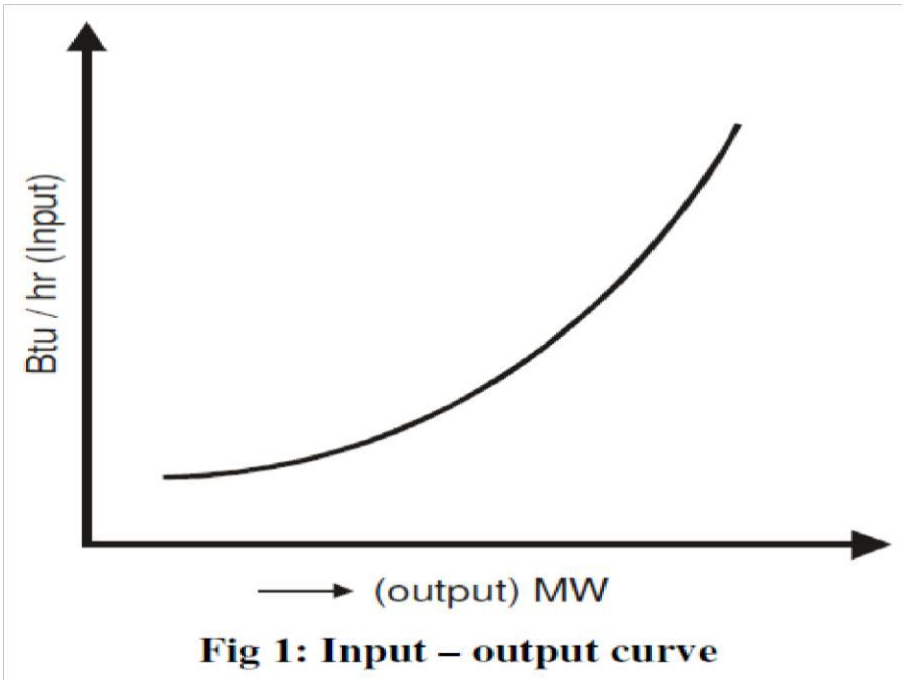


Fig 1: Input – output curve

HEAT RATE CURVE

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is called fuel –efficiency. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig .2

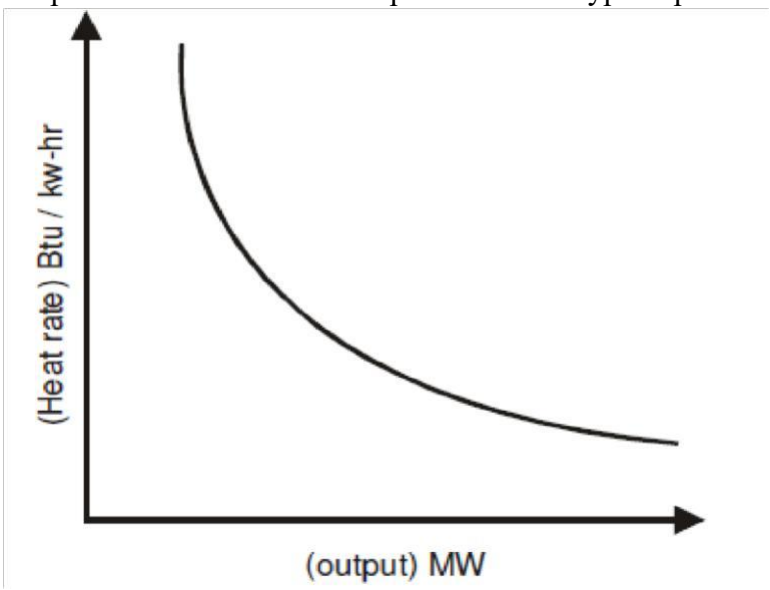


Fig .2 Heat rate curve.

INCREMENTAL FUEL RATE CURVE

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig 3

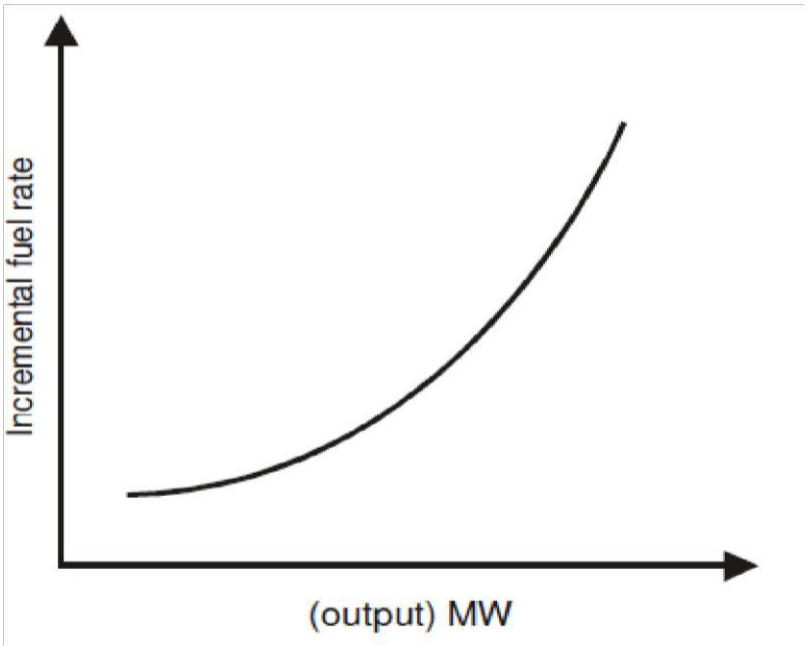


Fig 3: Incremental fuel rate curve

Incremental cost curve

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu or $ / Btu). The curve in shown in Fig. 4. The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

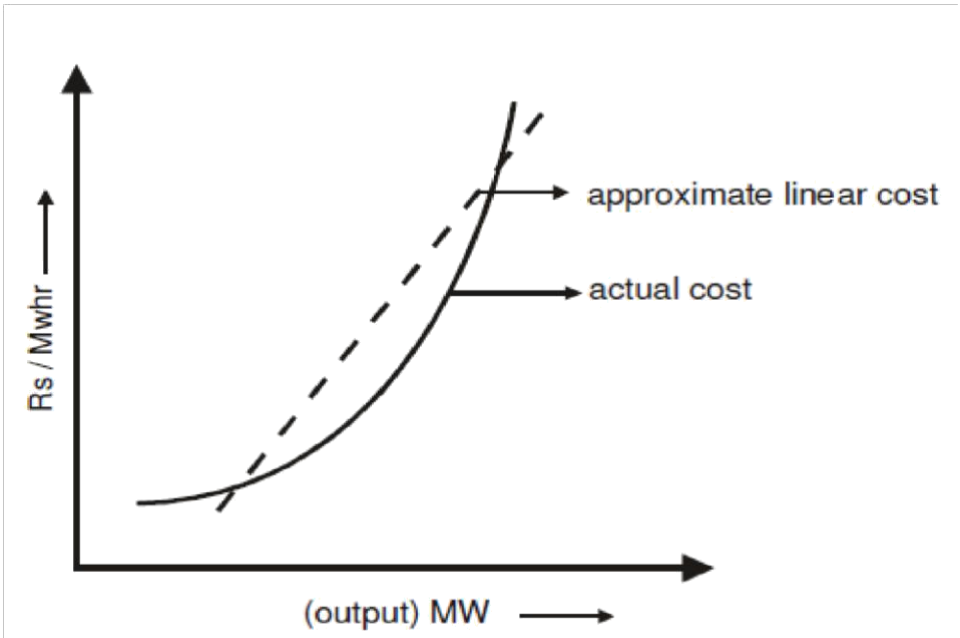


Fig 4: Incremental cost curve

In general, the fuel cost Fi for a plant, is approximated as a quadratic function of the generated output PGi.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad Rs / h$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \qquad Rs / MWh$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between PGmin, the minimum loading limit, below which it is technically infeasible to operate a unit and PGmax, which is the maximum output limit.

ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand PD. Consider a system with **n**g number of generating plants supplying the total demand PD. If Fi is the cost of plant ***i*** in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

where F_T = total cost.
P_{Gi} = generation of plant i.
P_D = total demand.

This is a constrained optimization problem, which can be solved by Lagrange‟ s method.

LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE The problem is restated below:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D = \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$£ = F_T + \lambda\left(P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial £}{\partial \lambda} = 0$$

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial £}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant Fi depends only on its own output PGi, hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \; ; \qquad i = 1 \ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1 \ldots\ldots n_g$$

The above equation is called the co-ordination equation. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand. From we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

It can be seen that l is dependent on the demand and the coefficients of the cost function.

Example 1.

The fuel costs of two units are given by

$F_1 = 1.5 + 20\ P_{G1} + 0.1\ P_{G1}^2$ Rs/h

$F_2 = 1.9 + 30\ P_{G2} + 0.1\ P_{G2}^2$ Rs/h

P_{G1}, P_{G2} are in MW. Find the optimal schedule neglecting losses, when the demand is 200 MW.

Solution:

$$\frac{dF_1}{dP_{G1}} = 20 + 0.2P_{G1} \quad \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 30 + 0.2P_{G2} \quad \text{Rs / MWh}$$

$$P_D = P_{G1} + P_{G2} = 200 \text{ MW}$$

For economic schedule

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \lambda$$

$$20 + 0.2\ P_{G1} = 30 + 0.2\ (200 - P_{G1})$$

Solving we get, $P_{G1} = 125$ MW

$P_{G2} = 75$ MW

$$\lambda = 20 + 0.2\ (125) = 45 \text{ Rs / MWh}$$

Example 2

The fuel cost in \$ / h for two 800 MW plants is given by

$F_1 = 400 + 6.0\ P_{G1} + 0.004\ P_{G1}^2$

$F_2 = 500 + b_2\ P_{G2} + c_2\ P_{G2}^2$

where P_{G1}, P_{G2} are in MW

(a) The incremental cost of power, λ is \$8 / MWh when total demand is 550MW. Determine optimal generation schedule neglecting losses.

(b) The incremental cost of power is \$10/MWh when total demand is 1300 MW. Determine optimal schedule neglecting losses.

(c) From (a) and (b) find the coefficients b_2 and c_2.

Solution:

a) $$P_{G1} = \frac{\lambda - b_1}{2c_1} = \frac{8.0 - 6.0}{2 \times 0.004} = 250 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 550 - 250 = 300 \text{ MW}$$

b) $$P_{G1} = \frac{\lambda - b_1}{2C_1} = \frac{10-6}{2\times 0.004} = 500 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 1300 - 500 = 800 \text{ MW}$$

c) $$P_{G2} = \frac{\lambda - b_2}{2c_2}$$

From (a) $$300 = \frac{8.0 - b_2}{2c_2}$$

From (b) $$800 = \frac{10.0 - b_2}{2c_2}$$

Solving we get $b_2 = 6.8$

$c_2 = 0.002$

ECONOMIC SCHEDULE INCLUDING LIMITS ON GENERATOR(NEGLECTING LOSSES)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\,(min)} \le P_{Gi} \le P_{Gi\,(max)} \; ; \; i = 1, \ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

Example 3

Incremental fuel costs in $ / MWh for two units are given below:

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 2.0 \ \$ / \text{MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.012P_{G2} + 1.6 \ \$ / \text{MWh}$$

The limits on the plants are P_{min} = 20 MW, P_{max} = 125 MW. Obtain the optimal schedule if the load varies from 50 – 250 MW.

Solution:

The incremental fuel costs of the two plants are evaluated at their lower limits and upper limits of generation.

At $P_{G(min)}$ = 20 MW.

$$\lambda_{1(min)} = \frac{dF_1}{dP_{G1}} = 0.01 \text{x } 20 + 2.0 = 2.2 \$ / \text{MWh}$$

$$\lambda_{2(min)} = \frac{dF_2}{dP_{G2}} = 0.012 \text{ x } 20 + 1.6 = 1.84 \ \$ / \text{MWh}$$

At $P_{G(Max)}$ =125 Mw

$$\lambda_{1(max)} = 0.01 \text{ x } 125 + 2.0 = 3.25 \ \$ / \text{MWh}$$

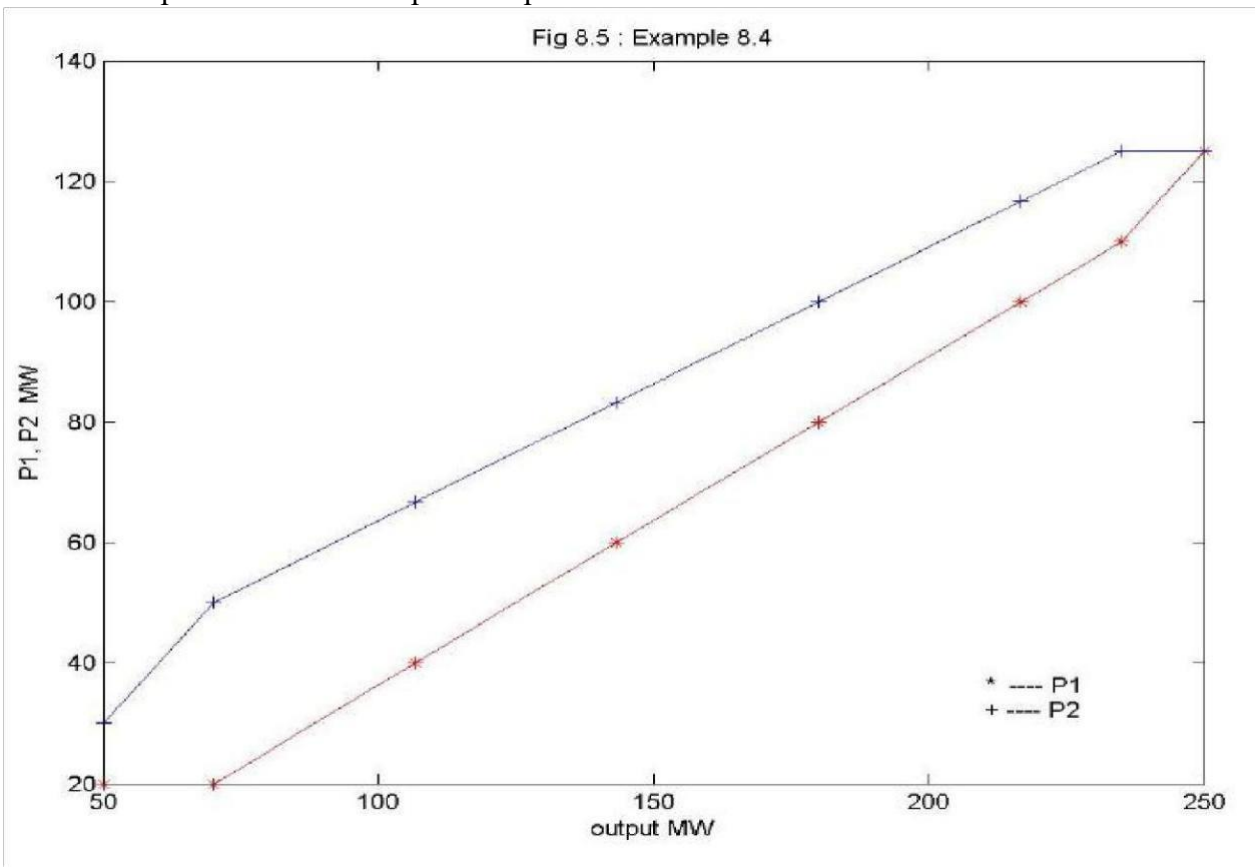
$$\lambda_{2(max)} = 0.012 \text{ x } 125 + 1.6 = 3.1 \ \$ / \text{MWh}$$

Now at light loads unit 1 has a higher incremental cost and hence will operate at its lower limit of 20 MW. Initially, additional load is taken up by unit 2, till such time its incremental fuel cost becomes equal to 2.2$ / MWh at PG2 = 50 MW. Beyond this, the two units are operated with equal incremental fuel costs. The contribution of each unit to meet the demand is obtained by assuming different values of l; When l = 3.1 $ / MWh, unit 2 operates at its upper limit. Further loads are taken up by unit 1. The computations are show in Table

Table Plant output and output of the two units

$\frac{dF_1}{dP_{G1}}$ \$/MWh	$\frac{dF_2}{dP_{G2}}$ \$/MWh	Plant λ \$/MWh	P_{G1} MW	P_{G2} MW	Plant Output MW
2.2	1.96	1.96	20^+	30	50
2.2	2.2	2.2	20^+	50	70
2.4	2.4	2.4	40	66.7	106.7
2.6	2.6	2.6	60	83.3	143.3
2.8	2.8	2.8	80	100	180
3.0	3.0	3.0	100	116.7	216.7
3.1	3.1	3.1	110	125*	235

For a particular value of l, PG1 and PG2 are calculated using (8.16). Fig 8.5 Shows plot of each unit output versus the total plant output.



For any particular load, the schedule for each unit for economic dispatch can be obtained.

Example 4.

In example 3, what is the saving in fuel cost for the economic schedule compared to the case where the load is shared equally. The load is 180 MW. Solution:

From Table it is seen that for a load of 180 MW, the economic schedule is PG1 = 80 MW and PG2 = 100 MW. When load is shared equally PG1 = PG2 = 90 MW. Hence, the generation of unit 1 increases from 80 MW to 90 MW and that of unit 2 decreases from 100 MW to 90 MW, when the load is shared equally. There is an increase in cost of unit 1 since PG1 increases and decrease in cost of unit 2 since PG2 decreases.

$$\text{Increase in cost of unit 1} = \int_{80}^{90}\left(\frac{dF_1}{dP_{G1}}\right)dP_{G1}$$

$$= \int_{80}^{90}(0.01P_{G1} + 2.0)dP_{G1} = 28.5\,\$/\text{h}$$

$$\text{Decrease in cost of unit 2} = \int_{100}^{90}\left(\frac{dF_2}{dP_{G2}}\right)dP_{G2}$$

$$= \int_{100}^{90}(0.012P_{G2} + 1.6)dP_{G2} = -27.4\,\$/\text{h}$$

Total increase in cost if load is shared equally = 28.5 – 27.4 = 1.1 $ / h

Hence the saving in fuel cost is 1.1 $ / h if coordinated economic schedule is used.

ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation. The mathematical formulation is now stated as

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such That $$\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$$

where P_L is the total loss.

The Lagrange function is now written as

$$£ = F_T - \lambda\left(\sum_{i=1}^{n_g} P_{Gi} - P_D - P_L\right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right) = 0\,;\ i = 1.....n_g$$

$$\frac{\partial £}{\partial \lambda} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \qquad \text{(Same as the constraint)}$$

Since $\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$, (8.27) can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda\frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}\right)$$

The term $\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant ***i***, $\mathbf{L_i}$. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i\,;\ i = 1.....n_g$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered. A rigorous general expression for the loss PL is given by

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn} + \Sigma_n P_{Gn} B_{no} + B_{oo}$$

where Bmn, Bno , Boo called loss – coefficients , depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values. A simpler expression is

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems has shown that the loss of accuracy is not significant if this approximation is used. An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses. In general, Bmn = Bnm and can be expanded for a two plant system as

$$P_L = B_{11} P_{G1} + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$$

Example 5

A generator is supplying a load. An incremental change in load of 4 MW requires generation to be increased by 6 MW. The incremental cost at the plant bus is Rs 30 /MWh. What is the incremental cost at the receiving end?

Solution:

$$\frac{dF_1}{dP_{G1}} = 30$$

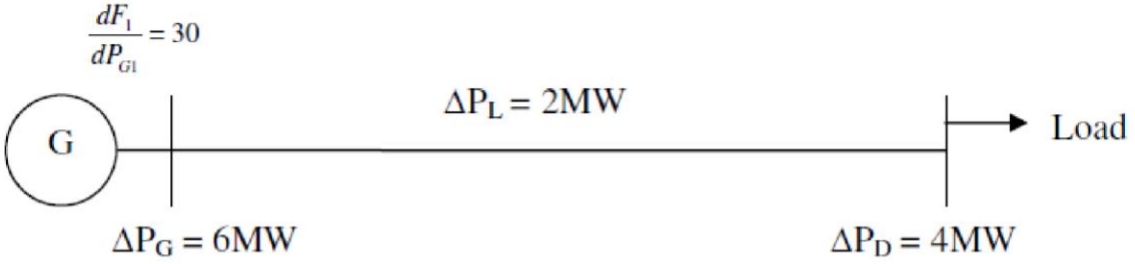


Fig ; One line diagram of example 5

$\Delta P_L = \Delta P_G - \Delta P_D = 2$ MW

λ at receiving end is given by

$$\lambda = \frac{dF_1}{dP_{G1}} \times \frac{\Delta P_G}{\Delta P_D} = 30 \times \frac{6}{4} = 45 \text{ Rs / MWh}$$

$$\text{or } \lambda = \frac{dF_1}{dP_{G1}} \times \frac{1}{1 - \frac{\Delta P_L}{\Delta P_G}} = 30 \times \frac{1}{1 - \frac{2}{6}} = 45 \text{ Rs / MWh}$$

Example 6

In a system with two plants, the incremental fuel costs are given by

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 20 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.015P_{G2} + 22.5 \text{ Rs / MWh}$$

The system is running under optimal schedule with $P_{G1} = P_{G2} = 100$ MW.

If $\frac{\partial P_L}{\partial P_{G2}} = 0.2$, find the plant penalty factors and $\frac{\partial P_L}{\partial P_{G1}}$.

Solution:

For economic schedule,

$$\frac{dF_i}{dP_{Gi}} L_i = \lambda ; \qquad L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} ;$$

For plant 2, $P_{G2} = 100$ MW

$$\therefore (0.015 \times 100 + 22.5)\frac{1}{1 - 0.2} = \lambda.$$

Solving, $\lambda = 30$ Rs / MWh

$$L_2 = \frac{1}{1 - 0.2} = 1.25$$

$$\frac{dF_1}{dP_{G1}} L_1 = \lambda \Rightarrow (0.01 \text{x} 100 + 20)\, L_1 = 30$$

$$L_1 = 1.428$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}}$$

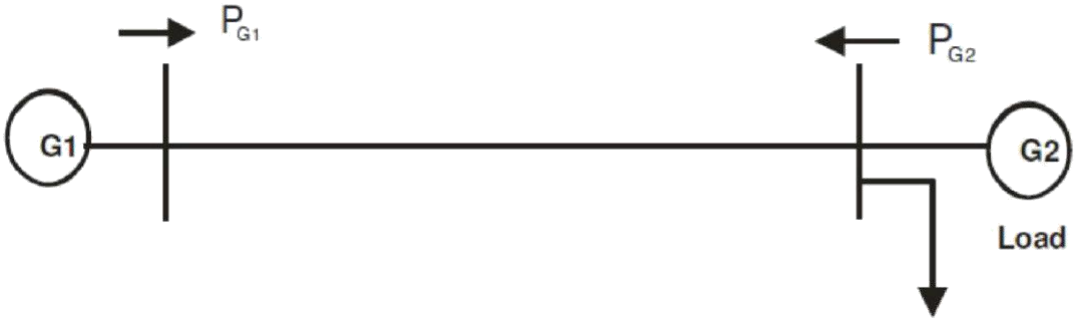
$$1.428 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} \quad ; \text{Solving} \quad \frac{\partial P_L}{\partial P_{G1}} = 0.3$$

Example 7

A two bus system is shown in Fig. 8.8 If 100 MW is transmitted from plant 1 to the load, a loss of 10 MW is incurred. System incremental cost is Rs 30 / MWh. Find P_{G1}, P_{G2} and power received by load if

$$\frac{dF_1}{dP_{G1}} = 0.02P_{G1} + 16.0 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.04P_{G2} + 20.0 \text{ Rs / MWh}$$



DERIVATION OF TRANSMISSION LOSS FORMULA

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

(i) All load currents have same phase angle with respect to a common reference (ii) The ratio X / R is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

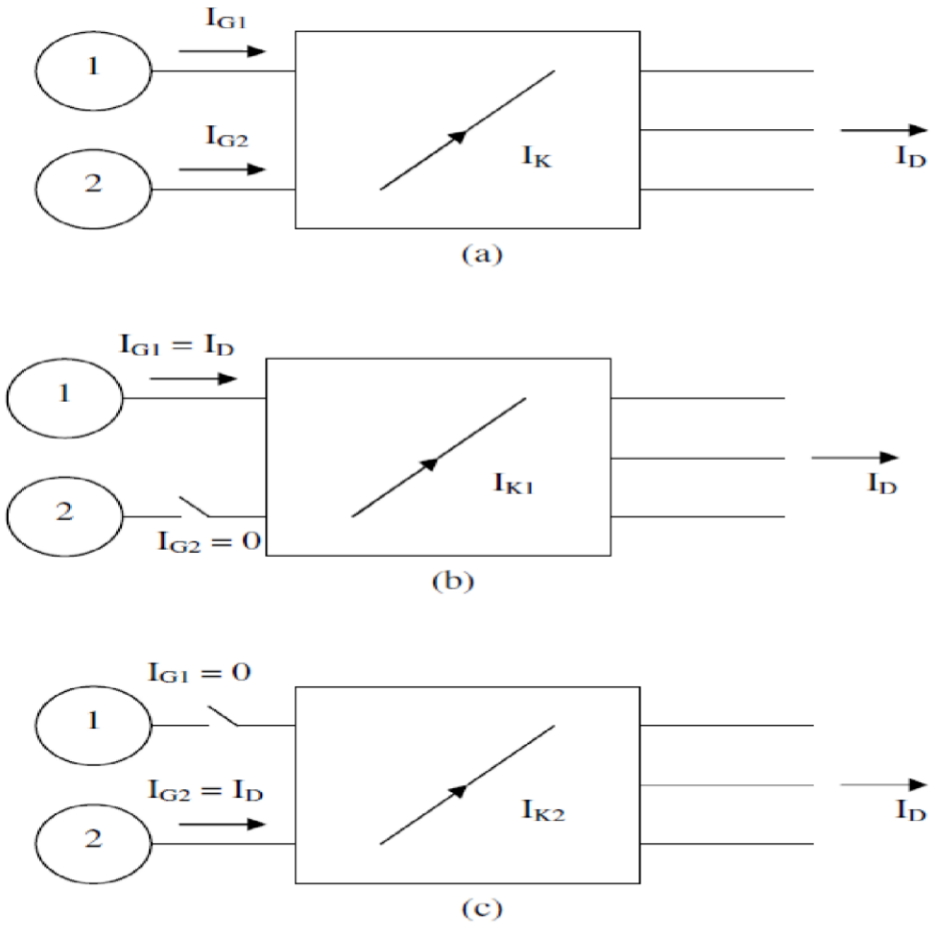


Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1} I_{G1} + N_{K2} I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}| \angle \sigma_1 \text{ and } I_{G2} = |I_{G2}| \angle \sigma_2 .$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= \begin{aligned} & N_{K1}{}^2|I_{G1}|^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}{}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right] \\ & + 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right] \end{aligned}$$

$$= N_{K1}{}^2|I_{G1}|^2 + N_{K2}{}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}{}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K + \frac{P_{G2}{}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}{}^2 R_K$$

$$P_L = P_{G1}{}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}{}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}{}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}{}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}{}^2 + \ldots\ldots + \frac{P_{Gn}{}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}{}^2 R_K + 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_k N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Example 8

Calculate the loss coefficients in pu and MW^{-1} on a base of 50MVA for the network of Fig below. Corresponding data is given below.

$I_a = 1.2 - j\ 0.4$ pu $\quad Z_a = 0.02 + j\ 0.08$ pu

$I_b = 0.4 - j\ 0.2$ pu $\quad Z_b = 0.08 + j\ 0.32$ pu

$I_c = 0.8 - j\ 0.1$ pu $\quad Z_c = 0.02 + j\ 0.08$ pu

$I_d = 0.8 - j\ 0.2$ pu $\quad Z_d = 0.03 + j\ 0.12$ pu

$I_e = 1.2 - j\ 0.3$ pu $\quad Z_e = 0.03 + j\ 0.12$ pu

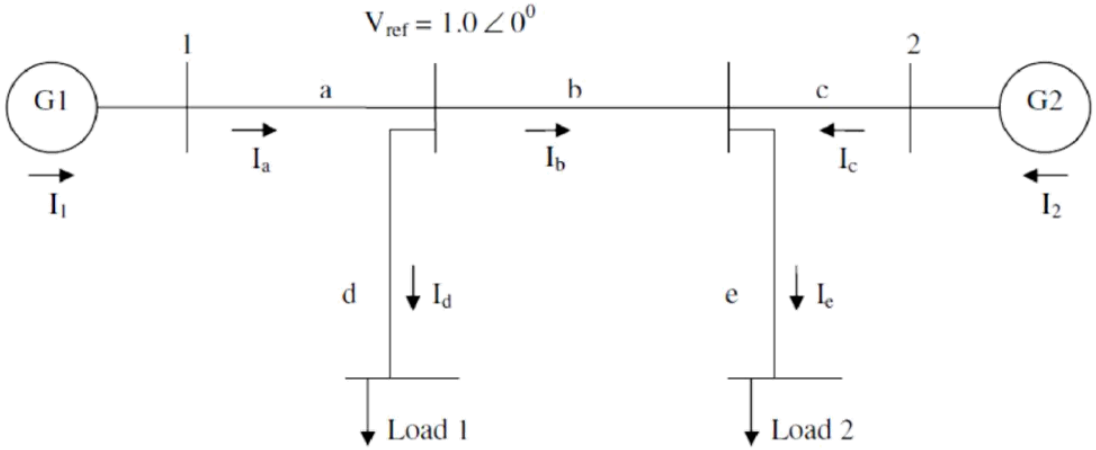


Fig : Example 8

Solution:

Total load current

$I_L = I_d + I_e = 2.0 - j\,0.5 = 2.061\ \angle -14.03^0 A$

$I_{L1} = I_d = 0.8 - j\,0.2 = 0.8246\ \angle -14.03^0\ A$

$$\frac{I_{L1}}{I_L} = 0.4; \qquad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

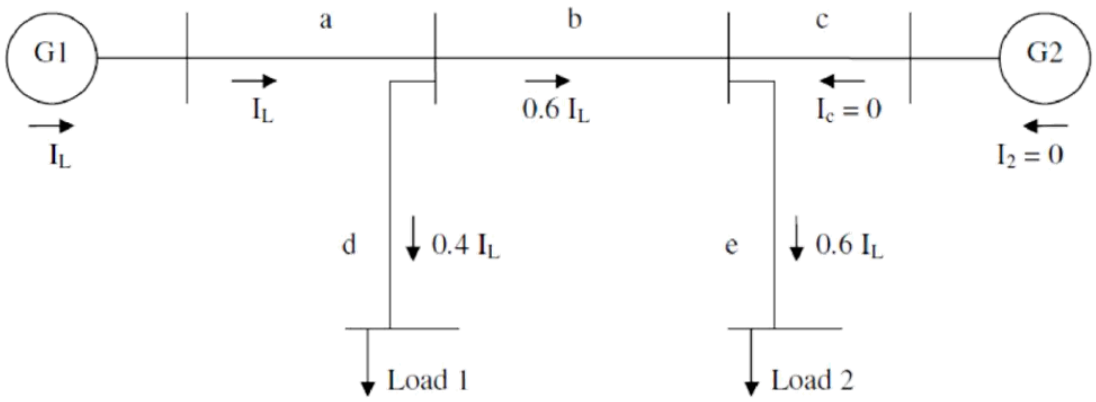


Fig a : Generator 1 supplying the total load

$$N_{a1} = \frac{I_a}{I_L} = 1.0;\ N_{b1} = \frac{I_b}{I_L} = 0.6;\ N_{C1} = 0;\ N_{d1} = 0.4;\ N_{e1} = 0.6.$$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

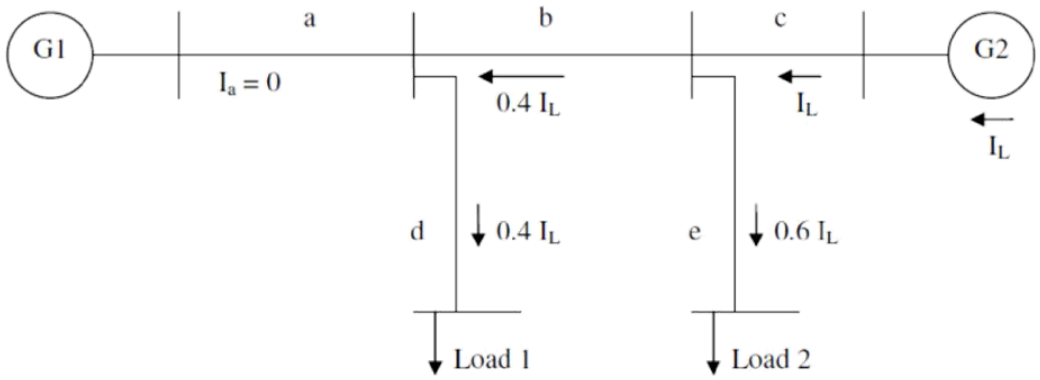


Fig b: Generator 2 supplying the total load

N_{a2} =0; N_{b2} = -0.4; N_{c2} = 1.0; N_{d2} = 0.4; N_{e2} = 0.6

From Fig 8.10, $V_1 = V_{ref} + Z_a I_a$

$= 1 \angle 0^0 + (1.2 - j\ 0.4)\ (0.02 + j0.08)$

$= 1.06 \angle 4.78^0 = 1.056 + j\ 0.088$ pu.

$V_2 = V_{ref} - I_b Z_b + I_c Z_c$

$= 1.0\ \angle 0^0 - (0.4 - j\ 0.2)\ (0.08 + j\ 0.32) + (0.8 - j\ 0.1)\ (0.02 + j\ 0.08)$

$= 0.928 - j\ 0.05 = 0.93\ \angle -3.10^0$ pu.

Current Phase angles

$$\sigma_1 = \text{angle of } I_1(=I_a) = \tan^{-1}\left(\frac{-0.4}{1.2}\right) = -18.43^0$$

$$\sigma_2 = \text{angle of } I_2(=I_c) = \tan^{-1}\left(\frac{-0.1}{0.8}\right) = -7.13^0$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

Power factor angles

$\phi_1 = 4.78^0 + 18.43 = 23.21^0; \cos\phi_1 = 0.92$

$\phi_2 = 7.13^0 - 3.10^0 = 4.03^0; \cos\phi_2 = 0.998$

$$B_{11} = \frac{\sum_K N_{K1}{}^2 R_K}{|V_1|^2 (\cos\phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

$= 0.0677$ pu

$= 0.0677$ x $\frac{1}{50} = 0.1354 \times 10^{-2}$ MW^{-1}

$$B_{12} = \frac{Cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)} \sum_K N_{K1} N_{K2} R_K$$

$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)}[-0.4\times0.6\times0.08+0.4\times0.4\times0.03+0.6\times0.6\times0.03]$$

$$= -0.00389 \text{ pu}$$

$$= -0.0078 \text{ x } 10^{-2} \text{ MW}^{-1}$$

$$B_{22} = \frac{\sum_K N_{K2}{}^2 R_K}{|V_2|^2(\cos\phi_2{}^2)}$$

$$= \frac{(-0.4)^2 0.08+1.0^2\times0.02+0.4^2\times0.03+0.6^2\times0.03}{(0.93)^2(0.998)^2}$$

$$= 0.056\text{pu} = 0.112\times10^{-2}\text{MW}^{-1}$$